Exercise 18

If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{1}{40}t\right)^2 \qquad 0 \le t \le 40$$

Find the rate at which water is draining from the tank after (a) 5 min, (b) 10 min, (c) 20 min, and (d) 40 min. At what time is the water flowing out the fastest? The slowest? Summarize your findings.

Solution

Take the derivative of the volume to get the rate that water flows out of the tank per minute.

$$\begin{aligned}\frac{dV}{dt} &= \frac{d}{dt}\left[5000\left(1 - \frac{1}{40}t\right)^2\right] \\ &= 5000(2)\left(1 - \frac{1}{40}t\right)^1 \cdot \frac{d}{dt}\left(1 - \frac{1}{40}t\right) \\ &= 10\,000\left(1 - \frac{1}{40}t\right)\left(-\frac{1}{40}\right) \\ &= -250\left(1 - \frac{1}{40}t\right)\end{aligned}$$

The rate that water is draining from the tank after $t = 5$ min is

$$\left.\frac{dV}{dt}\right|_{t=5} = -250\left[1 - \frac{1}{40}(5)\right] = -\frac{875}{4} = -218.75 \text{ gallons per minute.}$$

The rate that water is draining from the tank after $t = 10$ min is

$$\left.\frac{dV}{dt}\right|_{t=10} = -250\left[1 - \frac{1}{40}(10)\right] = -\frac{375}{2} = -187.5 \text{ gallons per minute.}$$

The rate that water is draining from the tank after $t = 20$ min is

$$\left.\frac{dV}{dt}\right|_{t=20} = -250\left[1 - \frac{1}{40}(20)\right] = -125 \text{ gallons per minute.}$$

The rate that water is draining from the tank after $t = 40$ min is

$$\left.\frac{dV}{dt}\right|_{t=40} = -250\left[1 - \frac{1}{40}(40)\right] = 0 \text{ gallons per minute.}$$

Notice that the rate that water flows out decreases linearly in time, so water flows out fastest at $t = 0$ and flows out slowest at $t = 40$.